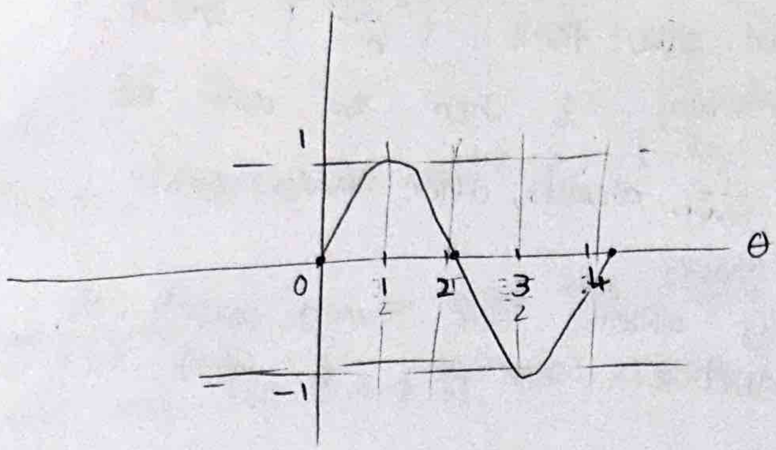
①

Exercise 2.2.2

1.



$$X_n = \sin\left(\frac{n\pi}{2}\right)$$

$\langle X_n \rangle_{0+2\lambda}$ where $\lambda \in \mathbb{N}$ is zero

$\langle X_n \rangle_{1+4\lambda}$: is positive where $\lambda \in \mathbb{N}$

$\langle X_n \rangle_{3+4\lambda}$ is negative where $\lambda \in \mathbb{N}$

$\therefore \langle X_n \rangle_n$ is strictly positive infinitely often, strictly negative infinitely often and zero infinitely often.

2. $$|X_n - 2| = \left|2 + \frac{(-1)^n}{n} - 2\right| = \left|\frac{(-1)^n}{n}\right|$$

The quotients: $$\frac{\left|\frac{(-1)^{n+1}}{n+1}\right|}{\left|\frac{(-1)^n}{n}\right|} = \frac{|(-1)|}{|n(n+1)|}$$

$\left.\left|\frac{(-1)^n}{n}\right|\right|_{n=1} = 1$, therfore if $|n(n+1)|^{\lambda}$ eventually equals 1000, the question will have been answered.

$$(n(n+1))^{\lambda} = 1000$$
$$\lambda = \frac{\log(1000)}{\log(n(n+1))}$$

3)

i. From question 2, we saw that $\left|\frac{(-1)^n}{n}\right|$ becomes less than $\frac{1}{1000}$ eventually, so then x_n will be greater than $2 - \frac{1}{1000}$ always, after that point.

ii. $P(x)$ holds infinitely often for $x = 2$, but not eventually? The sequence converges to ~~P(x)~~ $x_n = 2$

$(\exists N \in \mathbb{N})(\forall n \geq N)$ [x_n has property P]

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